Frequency Drift Characterization in Stable32

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Abstract

The characterization of frequency drift seems quite straightforward: Draw a line through a plot of the frequency data and observe its slope (or its mathematical equivalent, calculate and plot a least-squares linear fit to the data). But there are actually several ways to analyze frequency drift, based on various models, both linear and non-linear, using phase or frequency data, and optimized for various noise types. This document will describe the various methods for characterizing frequency drift that are supported by the Stable32 program for frequency stability analysis.

Introduction

It is often necessary to characterize the frequency drift inherent in a set of phase or frequency data as part of a stability analysis [2], [3]. That can be needed to quantify the frequency drift, and to model and remove it, before going on to analyze the noise and other properties of the source.

Statistical Estimation

Frequency drift characterization is an example of statistical estimation. Given a finite sample of phase or frequency data[1], the goal is, in the presence of noise, to estimate that parameter with the greatest precision (least variance) and without bias. We should therefore remember throughout this discussion that our result for frequency drift is only an estimate of the actual process.

Aging versus Drift

Frequency aging is defined as its change over time due to internal effects, while drift includes all causes including sensitivity to the external environment. The latter most importantly includes such accumulative effects as radiation and gas permeation. Aging measurements attempt to exclude external effects by operating the frequency source in a well-controlled environment, while drift measurements may subject the unit to external influences.

Preprocessing

As always, one should observe a plot of the frequency data, and must remove any outliers, before beginning a frequency drift analysis. Visual inspection of the data will largely determine whether a linear drift model is appropriate.

Linear Frequency Drift

A linear model is the most common way to describe frequency drift, using a single parameter

[1] One generally assumes that a statistical process is *stationary*, that its properties (e.g., average, variance, and drift) remain the same over time. But the average of a divergent random walk process does not. Fortunately, frequency drift measurements are stationary for all five common clock noises (see [8]), and more data therefore provides a higher-confidence estimate.

to quantify the change in frequency over the entire record length, the fractional frequency change per unit time[2]. In that case, the objective of a frequency drift analysis is simply to determine the value that best describes the rate of frequency change. What drift analysis method is "best" depends mainly on the underlying properties of the source noise[3].

• Non-Linear Drift Models

In some cases, a simple linear drift model is inappropriate to describe the behavior of a frequency source, and several non-linear models have been found useful for that purpose.

• Power Law Noise

The noise processes of a frequency source can be modeled by a set of *power law noises* whose power spectral densities are of the form $S(f) = h_\alpha \cdot f^\alpha$, where f is the Fourier frequency and α is the power law exponent from -2 (random walk FM), -1 (flicker FM), 0 (white FM or random walk PM), +1 (flicker PM) and +2 (white PM)[4], and h_α is the coefficient. The frequency drift must be determined in the presence of one or more of these noise types.

• Drift Analysis in Stable32

Elements of frequency drift analysis appear in several places in the Stable32 program as described below [1].

<u>Phase and Frequency Plots:</u> The Stable32 Plot function supports adding frequency drift lines to phase and frequency data plots.

The Frequency Plot Options dialog offers three frequency drift plot fit options (Line, Log and Diffusion), as well as polynomial and general function fits.

The Phase Plot Options dialog offers one frequency drift plot fit option (Quadratic), as well as polynomial and general function fits.

<u>Drift Function:</u> The Stable32 Drift function supports several models for frequency drift analysis and removal for both phase and frequency data as shown in Table 1.

Table 1. Drift Function Analysis Methods

Data	Method	Noise Model
Phase	Quadratic Fit	W PM
	Average of 2nd Differences	RW FM
	3-Point Fit	W & RW FM
	Greenhall 4-Point Fit	All
Freq	Linear Fit	W FM
	Bisection Fit	W & RW FM
	Log Fit	Stabilization
	Diffusion Fit	Diffusion
	Autoregression	AR(1)

Note that the slope value reported by the Drift function for the various fits is per tau interval, not per day, and must be adjusted accordingly.

For simulated phase and frequency data having frequency drift with negligible noise, all these drift fits will report the same value equal to that simulated. With significant noise, the values will differ somewhat, and the best estimation method will depend on the noise type. Greenhall grades them in Reference [5] where his 4-point fit ranks well for all noise types. Nevertheless, a linear least-squares fit to the frequency data is the most commonly used method, and is optimum for the white and flicker FM noise of many sources (e.g., Rb frequency standards).

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[2] This quantity is sometimes referred to as the *drift rate*, but we will generally use the simpler term *drift*.

[3] One may have a particular drift specification that determines its definition.

[4] In some cases the range of α is extended down to -4 (random run FM) and -3 (flicker walk FM).

The Log and Diffusion models are useful for frequency aging/drift that is stabilizing. The former implements the log model of MIL-PRF-55310, while the latter models a √t diffusion process.

Run Function: The Stable32 Run function supports analysis and removal of linear frequency drift during a stability run. After a run calculation, the frequency drift per day is reported per a linear least-squares fit to the frequency data, and there is a No Drift option to remove that frequency drift for the run.

The Run Plot Lines dialog includes a +1 slope noise line that can be used to fit an aging/drift characteristic in a stability plot.

Noise Function: The Stable32 Noise function includes the ability to include linear frequency drift in simulated phase and frequency data.

Scale Function: The Stable32 Scale function includes the ability to add or remove a certain linear slope to/from frequency data.

• Sigma-Tau Plots

The stability of a frequency source is commonly presented as a Sigma-Tau plot, a plot of log σ (e.g., the Allan deviation, ADEV) versus log τ (the averaging time), where the slope represents the type of power-law noise that applies. In addition, a slope of +1 indicates frequency drift, and the sigma value in a region with that slope can be used to determine the magnitude of the drift according to:

$\sigma_y(\tau) = (1/\sqrt{2})\cdot d\cdot\tau$, where d is the frequency drift.

For example, Figure 1 shows an ADEV plot for simulated frequency data having a drift of 1.0 per tau interval and negligible noise. As expected, the plot has a slope of +1 and a value of 0.707 at τ=1.0.

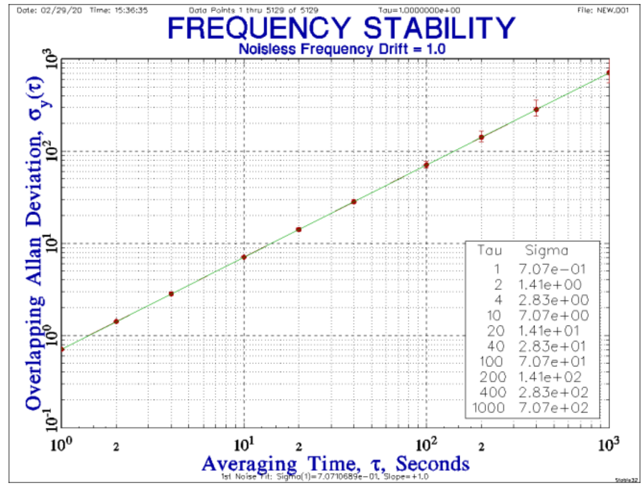

Figure 1. ADEV Plot of Frequency Drift

This method is generally useful only as an approximate way to quantify the frequency drift.

A more realistic example is the simulated 100 second rubidium frequency standard (RFS) data of Figure 2 which has a 1-second W FM noise level of 1×10^{-11}, a flicker floor of 2×10^{-13}, and a drift of 5×10^{-13}/day. In the corresponding ADEV plot of Figure 4, the expected τ^{+1} slope is 4.09×10^{-18} and the $\sigma_y(1)$ fit value above $\tau=2\times10^5$ seconds is 3.87×10^{-18}, providing a drift estimate of 4.73×10^{-13}/day, reasonably close to the simulated value.

The least-squares Linear fit to the frequency drift reports a value of 5.36×10^{-16} per 100 second tau interval, or 4.63×10^{-13} per day. Figure 3 shows the frequency residuals after removing this drift and averaging by a factor of 10.

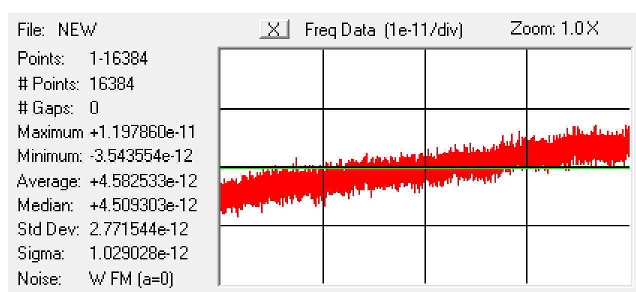

Figure 2. Simulated RFS Frequency Data

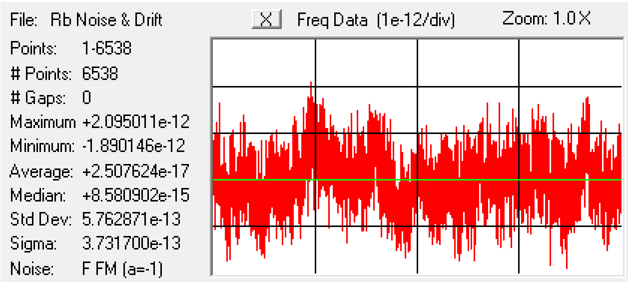

Figure 3. Flicker FM Frequency Residuals After Removal of Linear Frequency Drift and x10 Averaging

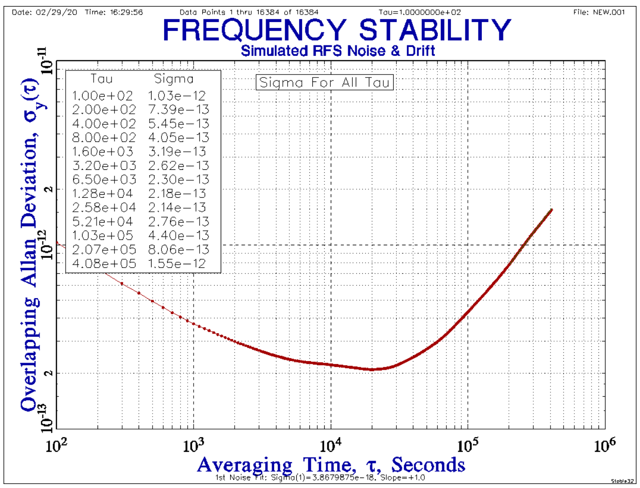

Figure 4. ADEV Plot of Simulated RFS

The Stable32 Drift function also reports a slope of 5.358791e-16 per 100 second tau interval for a linear fit to these frequency data, the optimum model for white FM noise, which corresponds to a drift of 4.63x10^{-13}/day. It reports nearly the same slope of 5.291951e-16 or 4.57x10^{-13}/day for a bisection fit, optimum for white and random walk FM noise. For the corresponding phase data, the quadratic fit slope is 5.324255e-16. The Run function reports a frequency drift value of 4.629995e-13/day per its linear least-squares fit.

• Simulations with Drift without Noise

Simulated clock data with frequency drift and differing types and levels of noise can be used to explore the various drift measures.

We first use the Stable32 Noise function to generate 16,384 frequency points with tau=1.0, Drift/Tau=1.0, and negligible noise, and observe that all the frequency drift methods report the same slope of exactly 1.000000 for both phase and frequency data.

• Simulations with White PM Noise

We then add a significant amount (1000) of white PM noise (see Figure 5).

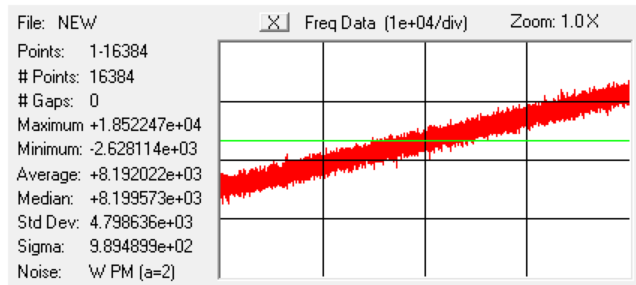

Figure 5. Simulated W PM Noise with Drift

Drift Type	Slope
Linear	1.000015e+00
Bisection	9.999918e-01
Quadratic	1.000000e+00
Avg of 2nd Diff	1.120271e+00
3-Point	9.999918e-01
Greenhall	1.000001e+00

The drift values are all reasonably close to the simulated value.

Note that the 3-Point phase method is equivalent to the Bisection frequency method, and their results are the same.

• Simulations with Flicker PM Noise

Next, we perform the same simulation with flicker PM noise (see Figure 6).

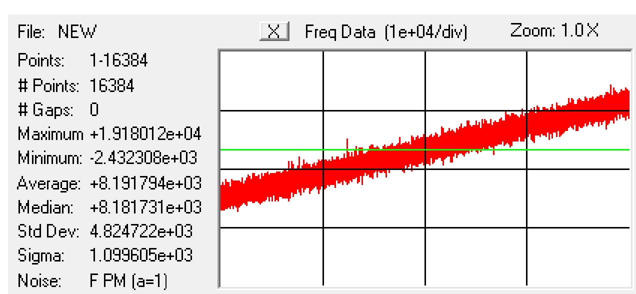

Figure 6. Simulated F PM Noise with Drift

Drift Type	Slope
Linear	1.000025e+00
Bisection	9.999664e-01
Quadratic	9.999729e-01
Avg of 2nd Diff	1.174360e+00
3-Point	9.999665e-01
Greenhall	9.999607e-01

The drift values are all reasonably close to the simulated value.

• Simulations with White FM Noise

Then we perform the same simulation with white FM noise (see Figure 7), and record the various slope values (your simulation results will, of course, vary):

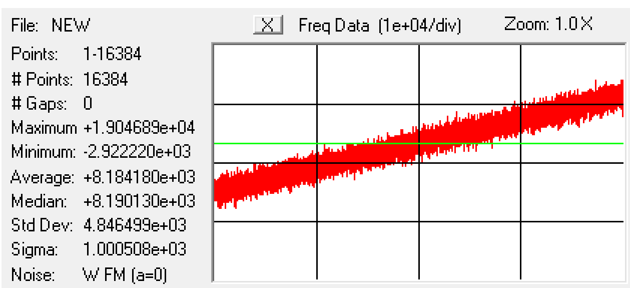

Figure 7. Simulated W FM Noise with Drift

Drift Type	Slope
Linear	1.002509e+00
Bisection	1.000740e+00
Quadratic	1.000531e+00
Avg of 2nd Diff	1.006156e+00
3-Point	1.000740e+00
Greenhall	1.001776e+00

There is no significant difference between the various methods, and all are very close to the simulated value.

• Simulations with Flicker FM Noise

Next, we perform the same simulation with flicker FM noise (see Figure 8) and tabulate those results.

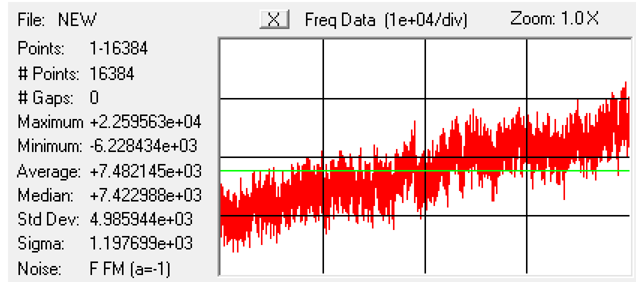

Figure 8. Simulated F FM Noise with Drift

Drift Type	Slope
Linear	9.197842e-01
Bisection	9.482337e-01
Quadratic	9.105587e-01
Avg of 2nd Diff	8.830378e-01
3-Point	9.482337e-01
Greenhall	8.887820e-01

As expected, there is considerably more variability in the results with the more divergent noise, and the Bisection/3-Point method does the best.

• Simulations with Random Walk FM Noise

Finally, we perform the simulation again with random walk FM noise (see Figure 9) and tabulate those results. We reduce the RW FM level to 30 for the purposes of this demonstration.

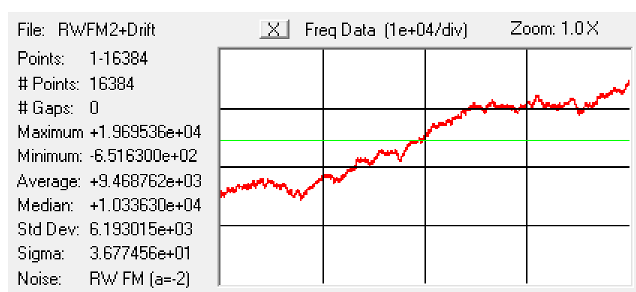

Figure 9. Simulated RW FM Noise with Drift

Drift Type	Slope
Linear	1.259368e+00
Bisection	1.396049e+00
Quadratic	1.417435e+00
Avg of 2nd Diff	1.169823e+00
3-Point	1.396049e+00
Greenhall	1.321140e+00

These results favor the Avg of 2nd Diff, which is recommended for RW FM noise.

• Non-Linear Frequency Drift Modeling

Stable32 provides two non-linear frequency drift models, plus the ability to fit a polynomial or arbitrary function. The log and diffusion models are used mainly to fit and remove frequency stabilization, and they are most appropriate when associated with some internal physical process such as mass redistribution in a quartz crystal oscillator or buffer gas pressure change in a rubidium frequency standard. An example of the latter is shown in Figure 10 where the green diffusion fit so closely follows the frequency record as to be nearly invisible.

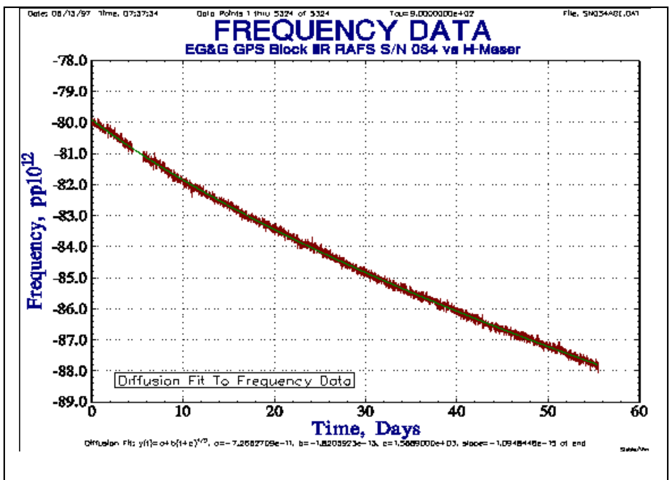

Figure 10. RFS Aging Data with Diffusion Fit

The numeric fit results can be used to estimate the linear frequency drift at a particular time.

• Confidence Intervals on Drift Estimates

The Stable32 software does not provide confidence intervals for its frequency drift estimates, nor is there a standard way for doing so. Some studies of this have used Monte Carlo simulations to determine the confidence of certain drift estimators in the presence of various power law noises [4], a technique much like shown here but with many more iterations so that their variance can be calculated.

For white FM noise and a linear least-squares linear frequency drift estimator, one can use ordinary statistical techniques to determine the uncertainty of that drift estimator, i.e., the confidence interval for the slope of a regression line.

Reference [5] shows how a Monte Carlo simulation was used to assess the confidence of the Average of 2nd Differences estimate of frequency drift.

Reference [6] shows that the uncertainty of the 3-Point estimator for frequency drift is a function of the Allan deviation of the drift-removed data at an averaging time equal to half the data length. That ADEV is estimated by extrapolating its log-log sigma-tau plot for the dominant power law noise type at long tau[5].

Reference [7] has a table with expressions for the error of frequency drift estimates for white, flicker and random walk FM noise. Those, like most such expressions, require knowledge of the Allan deviation at a tau a substantial fraction of the record length, a quantity that can be hard to obtain. Applying their expression[6] to the W FM noise of the Figure 3 RFS Linear frequency drift example results in error bounds of $4.63 \pm 0.02 \times 10^{-13}$/day, while their expression[7] for the F FM noise has a larger standard deviation of $\pm 0.27 \times 10^{-13}$/day. Combining those error variances results in a RFS drift estimate of $4.63 \pm 0.27 \times 10^{-13}$/day.

Reference [8] has tables with expressions for the errors of frequency and frequency drift estimates for difference and least-squares methods.

Reference [9] has a table showing expressions for the variances all of the Stable32 frequency

[5] The Total variance or Thêo1 may be used for that ADEV estimate.

[6] $2\sqrt{3} \cdot \sigma_y(\tau) / n^{3/2} \cdot X$, where $\sigma_y(\tau)=1\text{x}10^{-11}\tau^{-1/2}$, n=16,384, and τ=100.

[7] $3 \cdot \sigma_y(\tau) / n \cdot \tau \cdot \sqrt{2\ln 2}$, where $\sigma_y(\tau)=2\text{x}10^{-13}$, n=16,384, and τ=100.

drift estimators versus power law noise type, as reproduced in Appendix 1. For example, the standard deviation of a linear least squares frequency drift estimate (LSy) has an equivalent expression for F FM noise as that of Reference [7] (see footnote 7).

The procedure for estimating the confidence of a linear frequency drift determination could follow a process of (1) examining the frequency record (perhaps after averaging to reduce short-term noise) to choose an appropriate drift model and identify the long-term noise type, (2) examining the stability plot to determine the type and approximate magnitude of the long-term noise, (3) choosing the appropriate variance expression from the table in Appendix I, and (4) calculating the estimated drift variance.

The appropriateness of a non-linear (e.g., log or diffusion) frequency stabilization/drift determination is probably best judged by examining the frequency residuals after removing the modeled characterization. Examination of the frequency residuals after removal of linear drift will mainly show the noise.

• Conclusions

There is little practical difference between the various methods for frequency drift estimation, especially since 2 or 3-digit precision is usually sufficient. Similarly, there is little practical difference in their variances, or their response to various noise types. An exception to that would be an attempt at long term timekeeping based on knowledge of a relatively large frequency drift rate, but it is probably unrealistic to expect any such determination to remain valid anyway. Other concerns are that it can be quite hard to determine the variance of a frequency drift estimate, and that it is difficult to determine (or even define) the frequency drift in the presence of significant non-stationary random walk FM noise.

The intuitive and most widely used least-squares linear fit to the frequency data is generally the best choice, and, for a relatively long record length and most noise types, gives a reasonably precise result.

• References

1. *User Manual*, Stable32 Frequency Stability Analysis, Hamilton Technical Services, September 2008.
2. W.J. Riley, *Handbook of Frequency Stability Analysis*. You can buy a printed copy of this July 2008 book at Handbook, and download it as NIST Special Publication 1065. Please note that there is a typo in SP1065 Eq. 28 for TTOTVAR on p. 26: The tau exponent should be 2, not 3. And Eq. 9 is missing brackets around the inner summation – use Eq. 11 instead.
3. J.A. Barnes, "The Measurement of Linear Frequency Drift in Oscillators," *Proceedings of the 15th Annual Precise Time and Time Interval (PTTI) Meeting*, pp. 551-582, December 1983.
4. J.A. Barnes, "The Analysis of Frequency and Time Data," Austron, Inc., December 1991.
5. M.A. Weiss, D.W. Allan, and D.A. Howe, "Confidence on the Second Difference Estimation of Frequency Drift, A Study Based on Simulation," *Proceedings of the 1992 Frequency Control Symposium*, pp. 300-305, May 1992.
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9. C.A. Greenhall, "A Frequency-Drift Estimator and Its Removal from Modified Allan Variance," *Proceedings of the 1997*

IEEE International Frequency Control Symposium, pp. 428-432, May 1997.

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Appendix I

Table of Frequency Drift Estimator Variances

Table 1: Variance of frequency drift estimators. Names: $w4$ = 4-point w, LSx = least-squares quadratic fit to x, $x3$ = 3-point x, LSy = least-squares linear fit to y, LSz = least-squares constant fit to z, $\bar{y}2$ = 2-point $\bar{y}$. Each entry is to be multiplied by the factor on the right. The numbers in brackets are rankings within each noise type.

	w-discrete		x-discrete			
noise type	$w4$	LSx	$x3$	LSy	LSz or $\bar{y}2$	factor
white PM	$\frac{1250}{9}$[2]	90[1]	$24 f_h T$	$18 f_h T$		$h_2 \pi^{-2} T^{-5}$
flicker PM	74.84[1]	75[2]	$24 \ln(4.441 f_h T)$	$18 \ln(4.117 f_h T)$		$h_1 \pi^{-2} T^{-4}$
white FM	$\frac{200}{27}$[2]	$\frac{60}{7}$[4]	8[3]	6[1]	N	$h_0 T^{-3}$
flicker FM	10.900[2]	$\frac{25}{2}$[4]	$16 \ln 2$[3]	9[1]	$3 + 2 \ln N$	$h_{-1} T^{-2}$
rand.-wk. FM	$\frac{358}{135}$[3]	$\frac{20}{7}$[5]	$\frac{8}{3}$[4]	$\frac{12}{5}$[2]	2[1]	$h_{-2} \pi^2 T^{-1}$

where:

T = Length of phase record, x(t).

h_α = Magnitude of spectral density of fractional frequency variations, $S_y(f) = h_\alpha \cdot f^\alpha$, with f = Fourier frequency and α = power law exponent. It is given in Table B.2 of Reference [10] (IEEE Std 1139-1999). The h_α term contains the Allan variance expression for the noise.

Note: w4 is the Greenhall 4-point estimator.

From: C.A. Greenhall, "A Frequency-Drift Estimator and Its Removal from Modified Allan Variance," Proceedings of the 1997 IEEE International Frequency Control Symposium, pp. 428-432, May 1997.